

Introduction: Adaptive trials & sample size re-calculation

Marc Vandemeulebroecke

Part of the BBS training: *Advanced group-sequential and adaptive confirmatory clinical trial designs*

Basel, 13 Sep 2022

Learning objectives

Participants should understand:

- What are adaptive clinical trials
 - Major subtypes, distinctions, definitions
- Essential statistical methodology of adaptive trials
 - p-value combinations
 - conditional error functions
 - CRP principle
 - Conditional power and sample size adjustment

What are adaptive designs?

- There are many different types of adaptive trial designs
 - (Group sequential designs, early stopping)
 - Adaptive randomization
 - Adaptive dose escalation
 - Adaptive dose finding
 - Sample size re-estimation
 - Treatment arm selection
 - Enrichment designs
 - ...
- Various «schools» of adaptive designs have developed in parallel, depending on the application area

What are adaptive designs?

- Key distinctions:
 - Exploratory or confirmatory? → Confirmatory
 - Adaptations of which trial features? → Any
 - Using unblinded data? → Yes*
 - Predetermined adaptations or ad-hoc? → Both
 - Based on interim data or external information? → Both
- Excluded here:
 - Blinded design modifications (e.g. blinded sample size re-estimation; generally not controversial)
 - Bayesian designs (frequent in early development phases)
 - Response-adaptive randomization
- Our focus:
 - **Frequentist confirmatory adaptive designs**

Some definitions of adaptive designs

- Dragalin (PhRMA), 2006:
 - A multistage study design that uses accumulating data to decide how to **modify aspects of the study** without undermining the **validity** and **integrity** of the trial. [...] **preplanning**, as much as possible, based on **intended adaptations**.
- FDA draft guidance, 2010:
 - A study that includes a **prospectively planned** opportunity for **modification of one or more specified aspects** of the study design and hypotheses based on analysis of data (usually interim data) from subjects in the study.
- EMA reflection paper, 2007:
 - A study is called 'adaptive' if statistical methodology allows the **modification of a design element** [...] at an interim analysis with full **control of the type I error**.
- FDA guidance, 2019:
 - A clinical trial design that allows for **prospectively planned modifications to one or more aspects** of the design based on accumulating data from subjects in the trial.

Why adaptive designs

- In the 1980's, group sequential designs were introduced and grew popular. They provided a rigorous theory for early stopping but no other adaptations.
- In practice, however, adaptations of running trials were sometimes needed and done. Their impact on the inference was unclear and often ignored.
- **Quiz:** What is the maximal Type I error for a two-stage group-sequ. test with nominal level 5%* if n_2 is chosen in light of the observed first stage effect?
 - 5%? 8.2%? 11.5%?

Ignition: Bauer (1989)

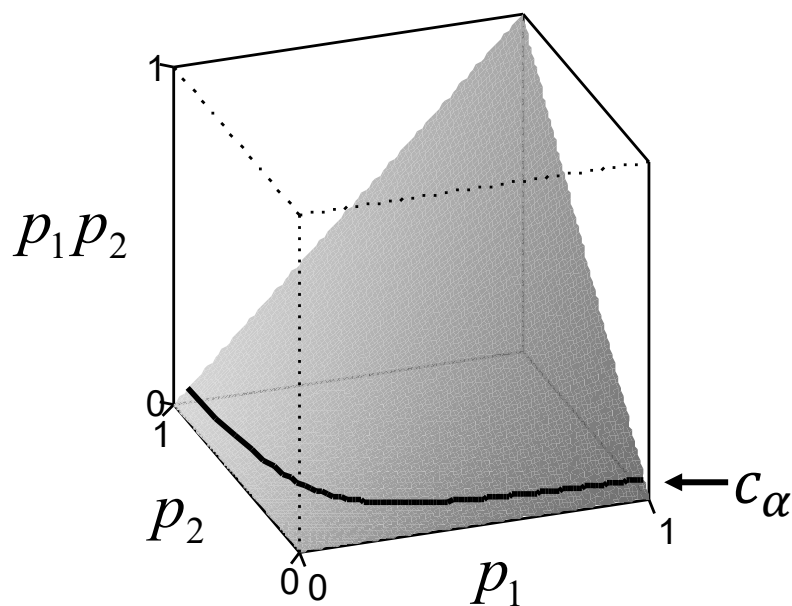
- Idea borrowed from meta-analysis (MA):
 - MA combines the inference from separate *trials*
 - Now: **combine the inference from separate stages of one trial**
 - This also allows adapting the second stage based on the first
- Method as well:
 - Take the product of the p-values from both trial stages
 - If $p_1 p_2$ is «too small» then reject H_0 .
 - **Quiz:** What is «too small»?
 - Hint: How are p_1 and p_2 distributed under the null hypothesis?

Fisher's product test

- $p_1, p_2 \sim_{H_0} U[0,1]$ iid
- $-2 \ln(p_1), -2 \ln(p_2) \sim_{H_0} \chi^2_2$ iid
- $-2(\ln(p_1) + \ln(p_2)) \sim_{H_0} \chi^2_4$
- Rejecting H_0 when $-2(\ln(p_1) + \ln(p_2)) \geq \chi^2_{4,1-\alpha}$ is a level α test
- Equivalently, rejecting H_0 when $p_1 p_2 \leq c_\alpha = \exp\left(-\frac{1}{2} \chi^2_{4,1-\alpha}\right)$

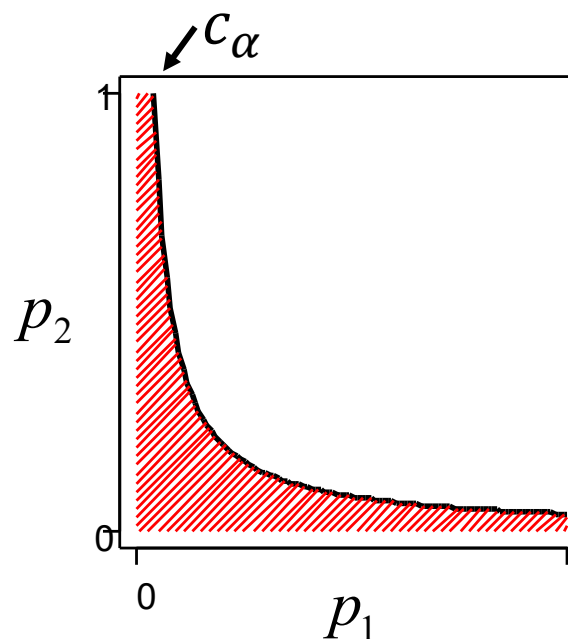
Let's look at it geometrically

- p-value combination



Reject if $p_1 p_2 \leq c_\alpha$

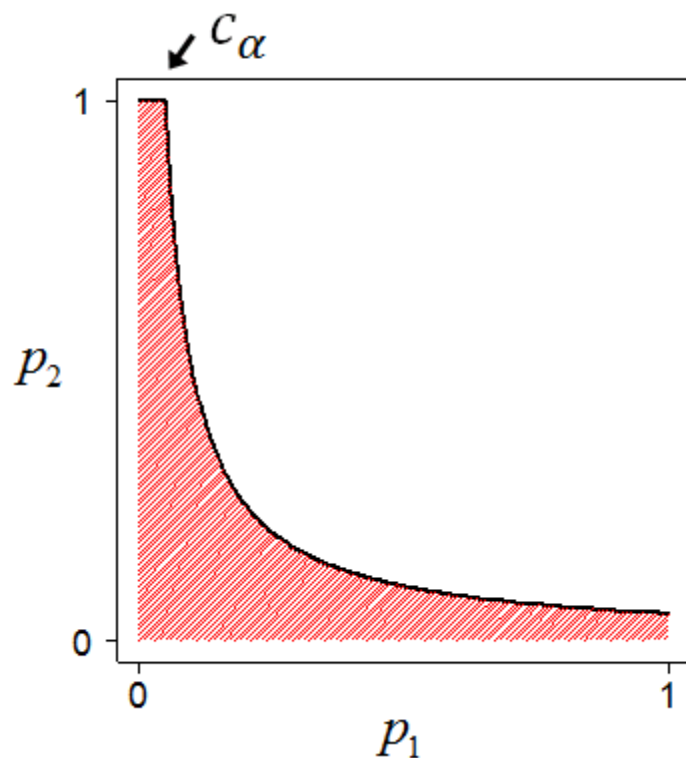
- Projection onto the plane



Reject if $p_2 \leq c_\alpha / p_1$

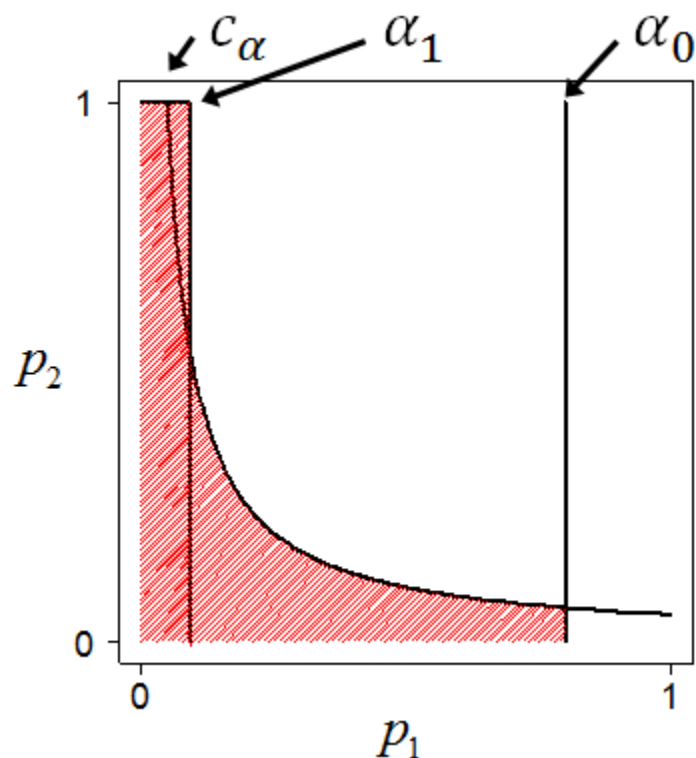
- **Quiz:** How large is the red area?

The conditional error function



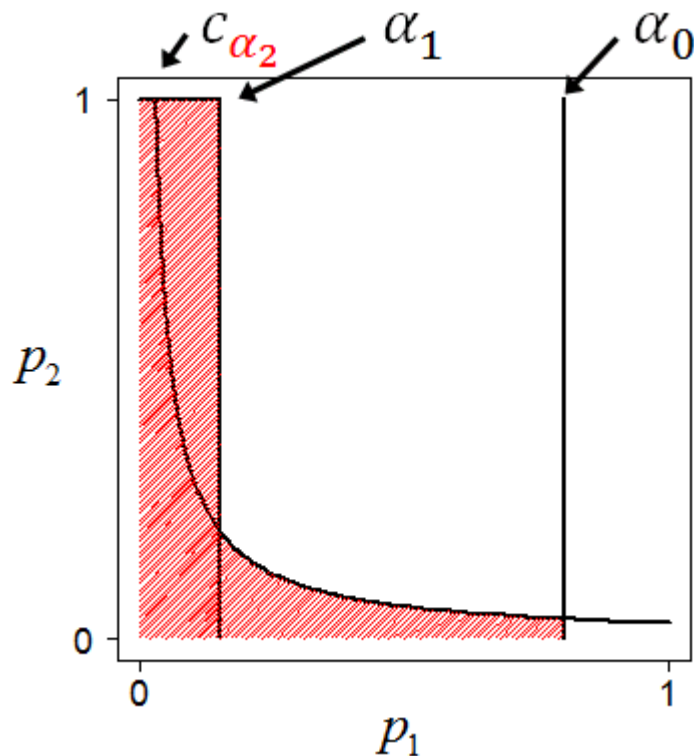
- Area of **rejection region**:
$$\int_0^{c_\alpha} 1 dp_1 + \int_{c_\alpha}^1 c_\alpha/p_1 dp_1 = c_\alpha - c_\alpha \ln(c_\alpha)$$
- But we know this must be α !
 - As $p_1, p_2 \sim_{H_0} U[0,1]$ iid, **areas correspond to probabilities**.
 - The rejection region has proba α .
- This **level curve defines** a level α test of H_0 . It is called a **conditional error function** (c.e.f.).
- Every **p-value combination** defines a **family of c.e.f.'s** that fills the unit square, and vice versa.

Early stopping



- Impose bounds α_1 and α_0
 - Assume $c_\alpha \leq \alpha_1 < \alpha_0$
 - $p_1 \leq \alpha_1 \rightarrow$ stop for efficacy
 - $p_1 > \alpha_0 \rightarrow$ stop for futility
 - Otherwise, perform second stage and reject H_0 if $p_2 \leq c_\alpha / p_1$
- Red area must remain α
 - $\alpha_1 + c_\alpha (\ln(\alpha_0) - \ln(\alpha_1)) = \alpha$

Change height of curve

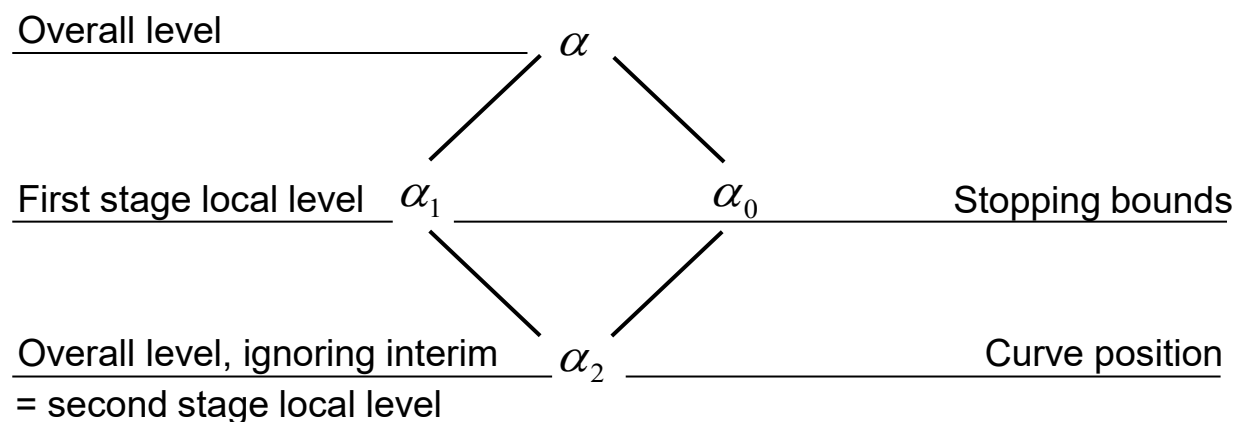


- Reject after second stage if $p_2 \leq c_{\alpha_2}/p_1$
 - This uses a different c.e.f. of the same family
 - The final test is performed at the **local level α_2**
- Red area must remain α

$$\alpha_1 + c_{\alpha_2} (\ln(\alpha_0) - \ln(\alpha_1)) = \alpha$$

The «alpha calculus»

- Four parameters are interdependent



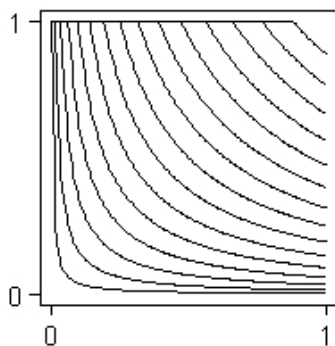
- Level condition: $\alpha_1 + c_{\alpha_2} (\ln(\alpha_0) - \ln(\alpha_1)) = \alpha$
- **Quiz:**
 - How would you specify a futility stop when control looks better?
 - How would you specify a «Pocock-type» test?

Inverse normal method & more

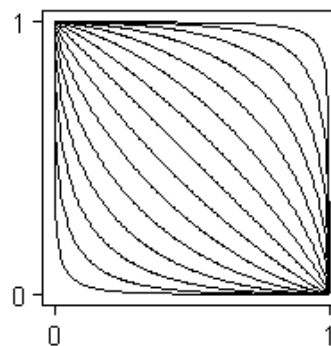
- Another natural way to combine p-values:

$$\frac{1}{\sqrt{2}} \left(\Phi^{-1}(1 - p_1) + \Phi^{-1}(1 - p_2) \right) \sim_{H_0} N(0,1)$$

- Same mechanism, with a different family of c.e.f.'s



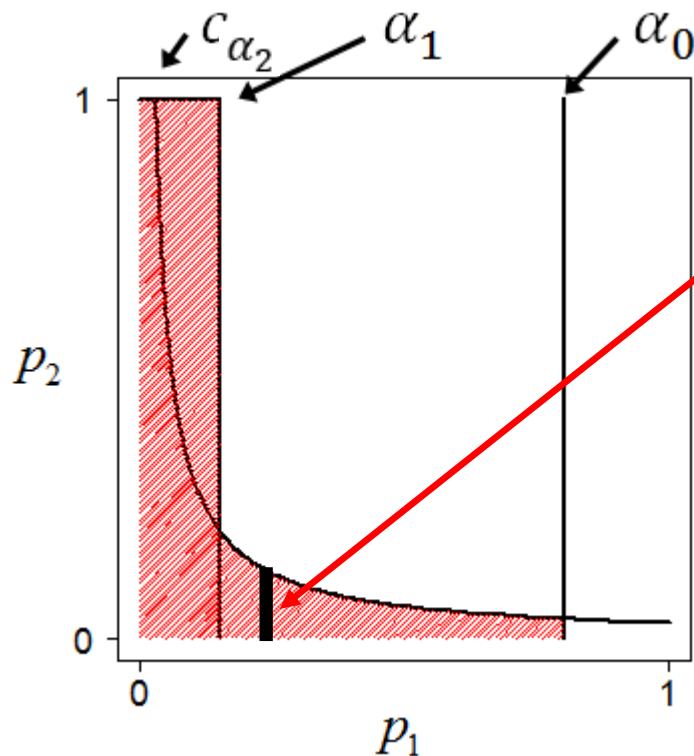
Product test



Inverse normal method (INM)

- In principle, any such family defines an adaptive test by this mechanism
 - In practice, mainly these two are used. And out of these, mostly INM.

How do trial adaptations fit into this?



- **This** height is the Type I error probability given the first stage data
- We could now change the second stage into any design that respects **this** level
- The resulting overall procedure remains a level α test

Why does this work?

- For continuously distributed test statistics based on separate stages, p_1, p_2 will generally be iid $U[0,1]$ **under H_0** even if the second stage is modified based on the interim analysis
- More generally, it still works if p_1, p_2 are only «p-clud»
 - $P_{H_0}(p_1 \leq u) \leq u$ and $P_{H_0}(p_2 \leq u | p_1) \leq u$ for all $u \in [0,1]$
- For more details on probabilistic foundations, see Brannath et al. 2012.

Conditional Rejection Principle (CRP)

- Start with a (classical) level α test
- At an IA, review the data and possibly external information
- No reason to adapt → Continue as planned
- Reason to adapt
 - Compute cond. Type I error of the pre-defined design:

$$P_{H_0}(\text{reject } H_0 \mid \text{interim data})$$

And choose (based on all info) a new design at **this** level to finish the trial

- This is a level α test, and the IA need not be preplanned

Conditional Rejection Principle (**CRP**)

- How could that new second-stage design look like?
 - Increase the remaining sample size (e.g., to achieve a desired **conditional power** → *see later*)
 - Note: Health authorities view sample size **reductions** more critically
 - Replace the second stage by another two-stage design → multistage designs by «**recursive combination**»
 - ...and more
- **Caveat**
 - Adaptations must not jeopardize **interpretability** of results or **credibility** of the trial!

Relation: Group sequ. ↔ adaptive

- Group sequential designs follow a **cumulative philosophy**: their test statistics are cumulative
- Adaptive designs follow a **stagewise philosophy**: they use stagewise inferences (test statistics, p-values)
 - However, the decision rules of adaptive designs **combine** the stagewise inferences – so overall **they do provide cumulative inference**
 - For example, Fisher's product test rejects H_0 if $p_2 \leq c_{\alpha_2}/p_1$
- The INM in particular reduces **exactly** to the group sequential test **if** no adaptations are done*. The test statistics, critical values and decision rules are identical.

→ *Next slide*

Relation: Group sequ. \leftrightarrow adaptive

- Test active vs. placebo with normally distr. endpoint
- Group sequential: $X_{ki} \sim N(\mu, \sigma^2)$ iid, $Y_{ki} \sim N(\nu, \sigma^2)$ iid
 - $k = 1, 2$ (stage); $i = 1, \dots, n_k$; σ^2 known
 - $n = n_1 + n_2$ total sample size per arm; $n_1 = n_2$ without loss of generality

- The Z-test:

- Overall: $Z = \sqrt{\frac{n}{2}} \frac{\bar{X} - \bar{Y}}{\sigma} \sim_{H_0} N(0, 1)$

- Per stage: $Z_k = \sqrt{\frac{n}{4}} \frac{\bar{X}_k - \bar{Y}_k}{\sigma} \sim_{H_0} N(0, 1)$; $p_k = 1 - \Phi(Z_k)$

- Group sequential: Using Z_1 and Z

- Inverse normal method:

- Combining p_1 and p_2 to $\frac{1}{\sqrt{2}} (\Phi^{-1}(1 - p_1) + \Phi^{-1}(1 - p_2)) = \frac{1}{\sqrt{2}} (Z_1 + Z_2) = Z$

Relation: Group sequ. \leftrightarrow adaptive

- The INM therefore generalizes the group sequ. test
 - Standard group sequential software can be used
- It is easily communicated with commonly used (Z-) statistics
- It is also the uniformly most powerful test if no adaptations are done
- All this is why the INM is often the **preferred method**

Weights

- More general version of the INM
 - Combine stagewise statistics using $w_1Z_1 + w_2Z_2$ instead of $\frac{1}{\sqrt{2}}(Z_1 + Z_2)$, with **weights** w_k
 - Weights can be freely chosen under the constraint $w_1^2 + w_2^2 = 1$
 - But they must be **prespecified** and remain **fixed** regardless of adaptations
 - Otherwise, the type I error may be inflated
 - Natural choice: $w_k = \sqrt{\frac{n_k}{n_1+n_2}}$
 - Then all patients carry equal weight, and again we have $w_1Z_1 + w_2Z_2 = Z$
 - The case $n_1 = n_2$ above was a special case of this

Efficiency vs. flexibility

- **Quiz:** What happens to the INM if we change the remaining sample size at the IA?
 - Not all patients carry equal weight → **inefficient**
- A curious debate
 - Tsiatis, Mehta (2003): “On the **inefficiency** of the adaptive design [...]”
 - Brannath et al. (2006): “On the **efficiency** of adaptive designs [...]”
- What do **you** think?
- In my view, trialists should weigh **efficiency** (power) against **flexibility** (adaptation)

Conditional power

- The **conditional power** is the power of the trial (at some alternative), given interim data
- Let's look at the inverse normal method
- Situation as before: $X_{ki} \sim N(\mu, \sigma^2)$ iid, $Y_{ki} \sim N(\nu, \sigma^2)$ iid
 - $k = 1, 2$ (stage); $i = 1, \dots, n_k$
 - $n = n_1 + n_2$ total sample size per arm
 - Denote $\vartheta = \frac{\mu - \nu}{\sigma}$

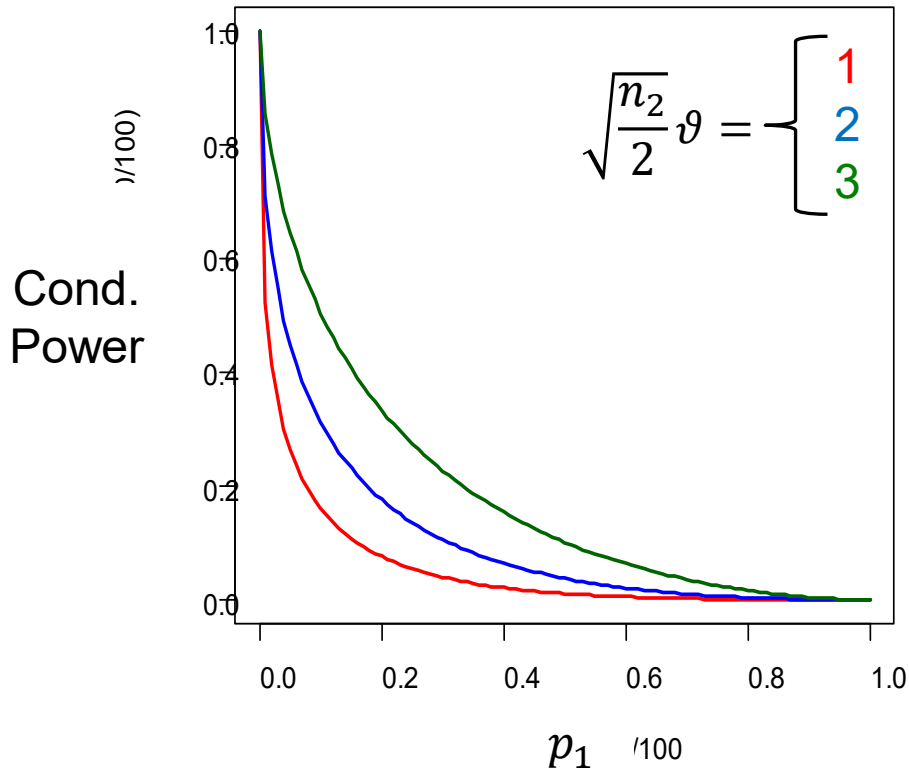
→ *Next slide*

Conditional power

- $$\begin{aligned} CP_{\vartheta} &= P_{\vartheta} \left(\frac{1}{\sqrt{2}} (\Phi^{-1}(1 - p_1) + \Phi^{-1}(1 - p_2)) \geq u_{\alpha} \mid p_1 \right) \\ &= P_{\vartheta} \left(\frac{1}{\sqrt{2}} (Z_1 + Z_2) \geq u_{\alpha} \mid Z_1 = z_1 \right) \\ &= P_{\vartheta} (Z_2 \geq \sqrt{2}u_{\alpha} - z_1) \\ &= P_{\vartheta} \left(Z_2 - \sqrt{\frac{n_2}{2}}\vartheta \geq \sqrt{2}u_{\alpha} - z_1 - \sqrt{\frac{n_2}{2}}\vartheta \right) \\ &= 1 - \Phi \left(\sqrt{2}u_{\alpha} - z_1 - \sqrt{\frac{n_2}{2}}\vartheta \right) \end{aligned}$$

Conditional power

- Properties



- Conditional power
 - Increases with n_2
 - Increases with ϑ
 - Decreases for increasing p_1

Conditional power

- Common applications

- Stopping for futility if CP_{ϑ} is «too small» (e.g. below 20%)
- Adjusting the second stage size to achieve a desired CP_{ϑ} (e.g. 90%)

In the example, solve $0.9 = 1 - \Phi\left(\sqrt{2}u_{\alpha} - z_1 - \sqrt{\frac{n_2}{2}}\vartheta\right)$ for n_2

Conduct the second stage and perform the final inference as planned through the adaptive design

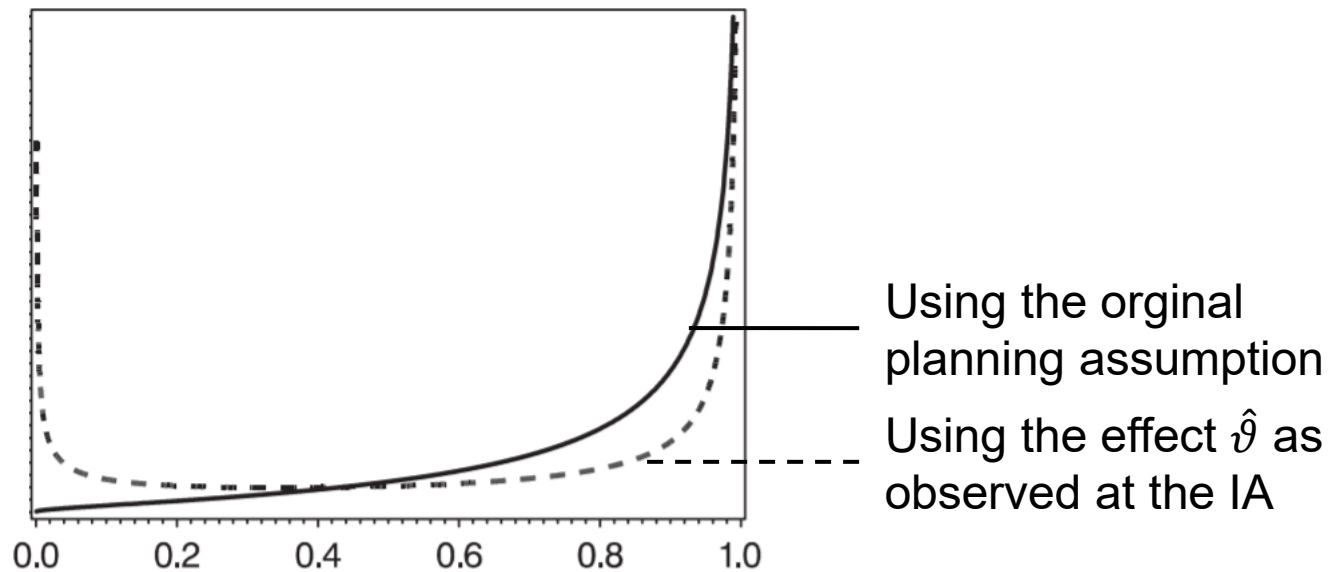
- **Quiz:** What ϑ would you use in CP_{ϑ} ?

Conditional power

- Several options for ϑ in CP_{ϑ}
 - The originally assumed effect size for sample size calculation (minimally clinically relevant effect – should not have changed!)
 - The effect size $\hat{\vartheta}$ as observed at the interim analysis (hoping that this comes closer to the «truth»)
 - **Caution:** Interim estimates such as $\hat{\vartheta}$ are notoriously volatile! → *Next slide*
 - Averaging across several choices
 - Weighted average of originally assumed and observed effect size
 - Integrating over some distribution for ϑ («**predictive power**»)

Conditional power

- Using the interim effect estimate is risky
 - Because we rely **doubly** on little data: through z_1 and through $\hat{\vartheta}$
 - The density of CP_{ϑ} tends towards extremes if we use $\hat{\vartheta}$



The «Constrained Promising Zone» (CPZ) Approach

- A recent proposal for a more refined use of conditional power to re-calculate the sample size
 - Builds upon the previously proposed «Promising Zone» approach by Mehta and Pocock (2011) which had been shown to be (overly) conservative (Glimm 2012, Jennison and Turnbull 2015)
- Idea: *Boost the sample size within reasonable limits when the interim treatment effect appears «promising»*

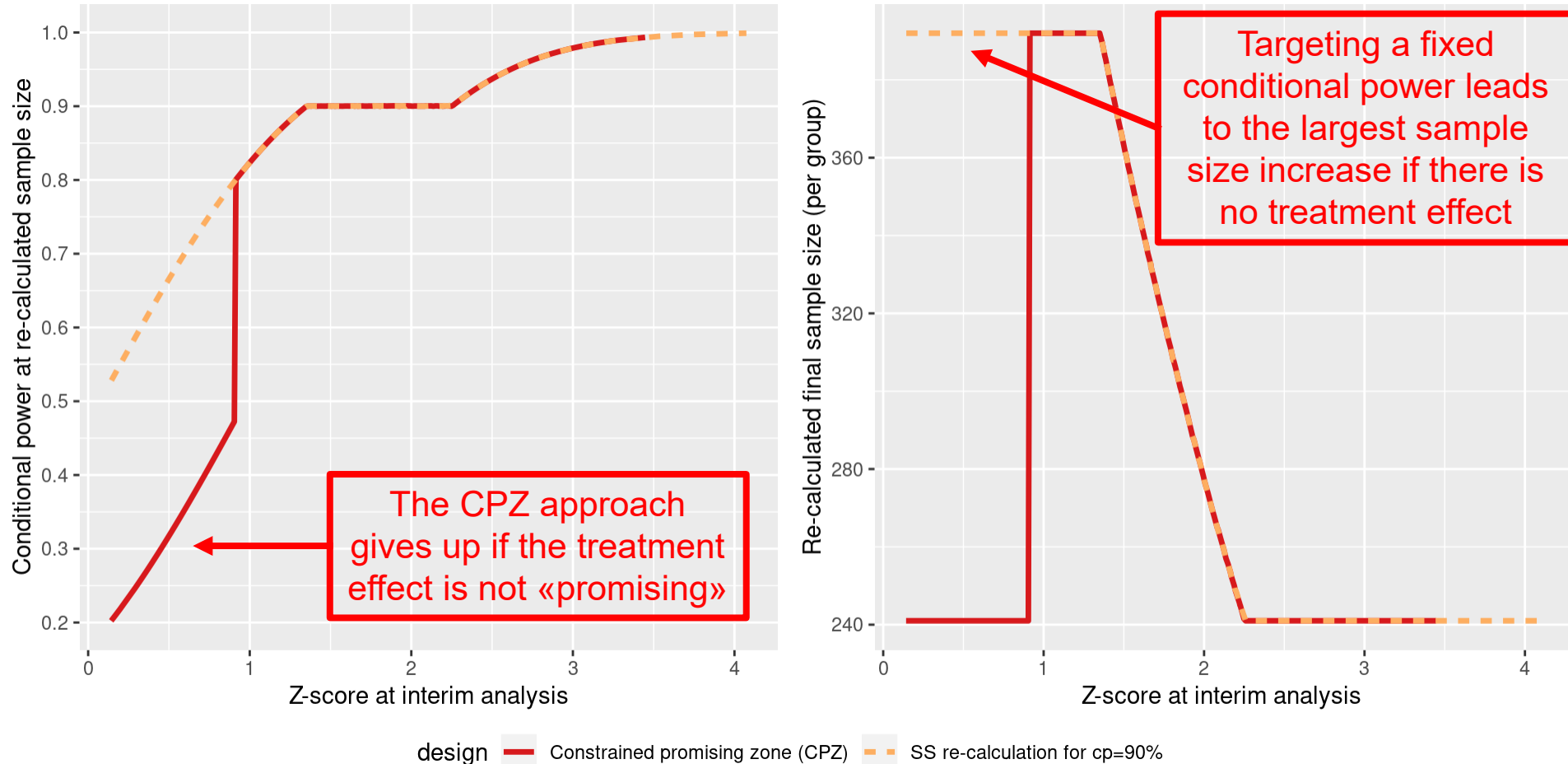
The «Constrained Promising Zone» (CPZ) Approach

- Concretely, pre-specify:
 - Impose limits to allowed total sample size per arm: n_{min}, n_{max}
 - Set smallest clinically meaningful effect size ϑ_{min} , and smallest / largest desired conditional power at this point: CP_{min}, CP_{max}
 - Choose a combination test, e.g. INM with $w_1 = \sqrt{\frac{n_1}{n_{min}}}$, $w_2 = \sqrt{\frac{n_{min}-n_1}{n_{min}}}$
- Then re-calculate the sample size at the IA:
 - If n^* exists between n_{min} and n_{max} such that $CP_{\vartheta_{min}}(z_1, n^*) = CP_{max}$, then set the total sample size (per arm) to n^*
 - Otherwise, if $CP_{\vartheta_{min}}(z_1, n_{max}) \geq CP_{min}$, then set it to n_{max}
 - Finally, otherwise, set it to n_{min} because the IA is not «promising»

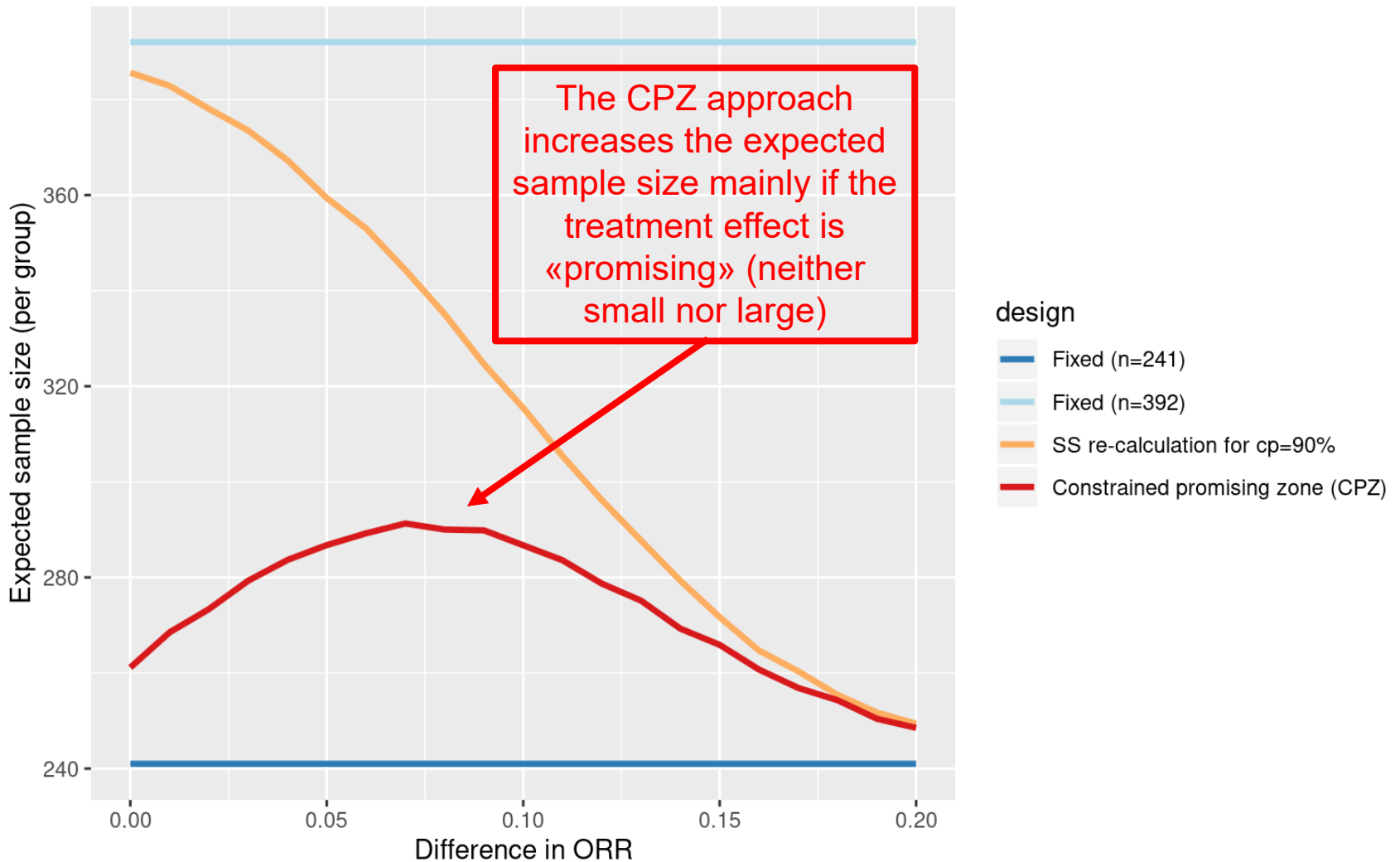
The «Constrained Promising Zone» (CPZ) Approach - Example

- 1:1 randomization with Overall Response Rate (ORR) as primary endpoint
 - ORR=20% on Control; Drug increases this by 10-13%
 - 2.5% significance level (one-sided)
 - $n_1 = 120$
 - $n_{min} = 241, n_{max} = 392$ (90% power for $\Delta=13%$ and $\Delta=10%$, resp.)
- Compare two approaches
 - Sample size increase for a conditional power of 90% (if true $\Delta=10%$)
 - CPZ design with $CP_{min} = 80%$, $CP_{max} = 90%$
- Corresponding R-code is in this vignette
 - [Simulation of a Trial with a Binary Endpoint and Unblinded Sample Size Re-Calculation with rpact](#)

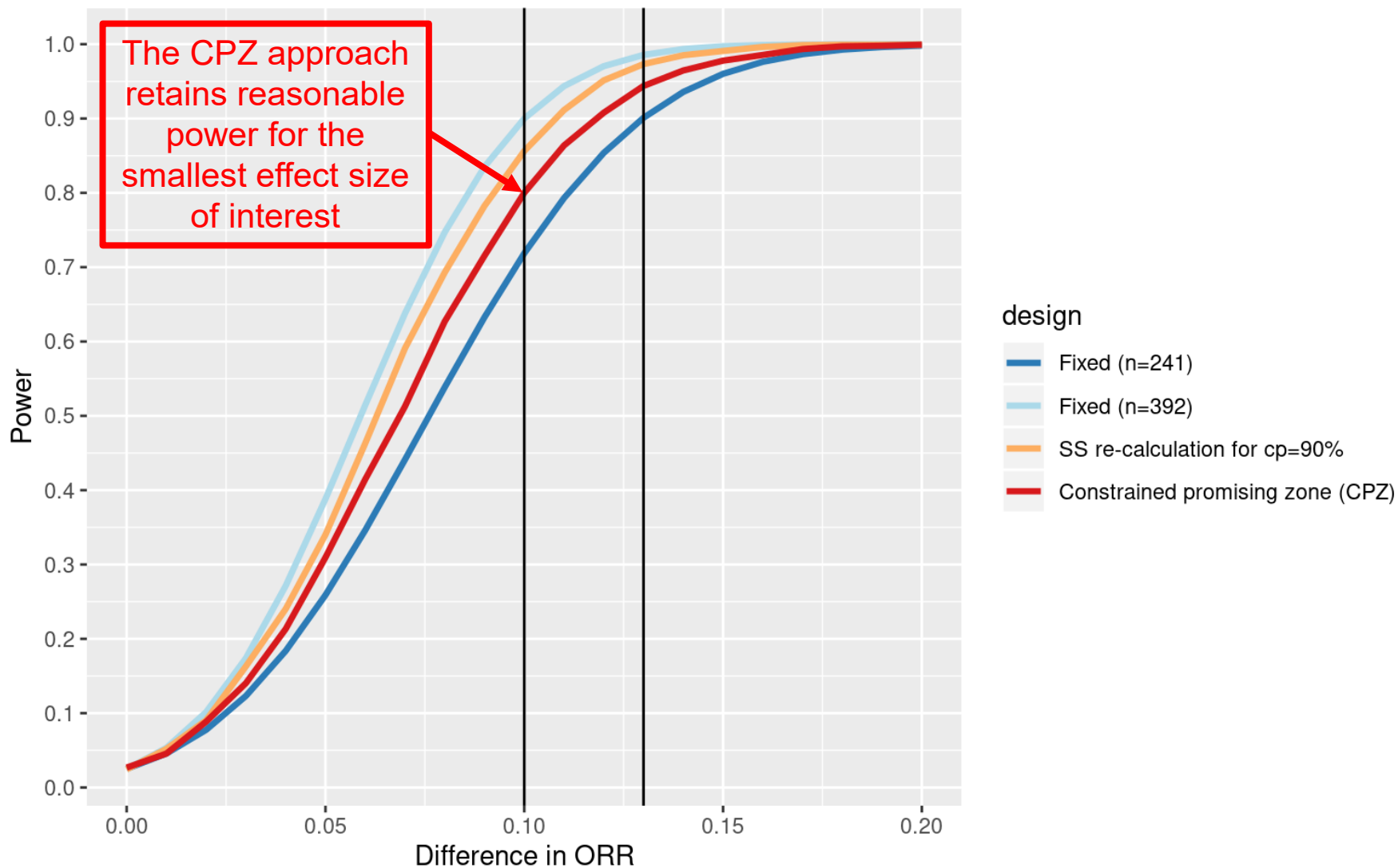
Cond. power and total sample size depending on the interim Z-score



Expected sample size depending on effect size



Power depending on effect size



Regulatory guidance on unblinded sample size adaptation

EMA guidance on adaptive designs 2007

- The option to reassess sample size in an ongoing trial should **not be seen as a substitute for careful planning**. The relevance of a particular size of treatment effect should be discussed at the planning stage of the trial and not deferred to the point where interim results are already available.
- Whenever possible, methods for blinded sample size reassessment that properly control the type I error should be used [...]. In cases where sample size needs to be reassessed based on unblinded data, **sufficient justification** should be made.

FDA adaptive designs guidance 2019

- [such designs] might be used when there is considerable **uncertainty about the true treatment effect size**.
- [...] to appropriately **control the Type I error** [...and] **prospective planning** [...of] the statistical hypothesis testing method [...and] the rule governing the sample size modification.
- [...] additional challenges in maintaining **trial integrity** [...]

Our recommendations for unblinded sample size adaptation

- Approach is **accepted** by health authorities, but **more justification** is needed than for blinded sample size adaptation
- Main application: Considerable **uncertainty about the size of the treatment effect** and **reluctance to fund a group-sequential trial** powered to the smallest clinically relevant effect size
 - «*Start small and invest more resources if results look promising*»
- Extensive literature of such designs versus «more efficient» group-sequential designs
 - E.g., Liu et al, 2018: «*...under reasonable decision rules for increasing sample size [...] there is little or no loss of efficiency for the adaptive designs in terms of unconditional power. The two approaches, however, have very different conditional power profiles.*»
- Extensive **clinical trials simulations** and comparisons to group-sequential designs are highly recommended
 - Can also help to explore potential bias in estimation
 - **rpact** can produce median unbiased estimators and other inference adjusted for the adaptive design

Final thoughts on adaptive designs

- **Allowance for adaptations** of the trial design without inflating type I error
 - Adaptations should be **pre-planned in most circumstances**
 - ...but can be occasionally be used to react to unforeseen circumstances
- Can be **extended** to multi-arm and enrichment designs (covered later)
- Adaptive designs are **more complicated** than fixed or group-sequential designs in terms of **trial planning, logistics**, and **regulatory requirements** to ensure trial integrity and avoid operational bias
- Two attitudes:
 - The **social event trial**: «Let's come together, let's see and then adapt until significance» (Koch 2006)
 - **Much better**: «A multistage study design that uses accumulating data to decide how to modify aspects of the study without undermining the validity and integrity of the trial.» (Dragalin 2006)
- **Adaptive designs are not a remedy for sloppy planning!**

References (1)

Reference book

- Wassmer, Brannath (2016): Group Sequential and Confirmatory Adaptive Designs. Springer

Reviews

- Todd (2007): A 25-year review of sequential methodology in clinical studies. *Statistics in Medicine* 26, 237–252
- Vandemeulebroecke (2008): Group Sequential and Adaptive Designs – A Review of Basic Concepts and Points of Discussion. *Biometrical Journal* 50, 541–557
- Bauer et al. (2016): Twenty-five years of confirmatory adaptive designs: opportunities and pitfalls. *Statistics in Medicine* 35, 325–347

Regulatory guidance

- EMA (2007). Reflection paper on methodological issues in confirmatory clinical trials planned with an adaptive design. European Medicines Agency, CHMP/EWP/2459/02
- FDA (2010): Draft guidance for industry. Adaptive design clinical trials for drugs and biologics. Rockville, MD
- FDA (2019): Guidance for industry. Adaptive designs for clinical trials of drugs and biologics. Silver Spring, MD
- Brannath et al. (2010): Comments on the Draft Guidance on “Adaptive Design Clinical Trials for Drugs and Biologics” of the U.S. Food and Drug Administration. *Journal of Biopharmaceutical Statistics* 20, 1125–1131

Discussion & interpretation

- Dragalin (2006): Adaptive Designs: terminology and classification. *Drug Information Journal* 40, 425–435
- Koch (2006): Confirmatory clinical trials with an adaptive design. *Biometrical Journal* 48, 574–585. Rejoinder pp. 616–622
- Burman, Sonesson (2006): Are flexible designs sound? (with discussion). *Biometrics* 62, 664–683
- Vandemeulebroecke (2008): *see above*

References (2)

Methodological contributions

- Fisher (1932): Statistical methods for research workers. Oliver & Boyd, London
- Marcus, Peritz, Gabriel (1976): On closed testing procedures with special reference to ordered analysis of variance. *Biometrika* 63, 655–660.
- Bauer (1989): Multistage testing with adaptive designs (with discussion). *Biometrie und Informatik in Medizin und Biologie* 20, 130–148
- Bauer, Köhne (1994): Evaluation of experiments with adaptive interim analyses. *Biometrics* 50: 1029-1041. Correction in *Biometrics* 52 (1996): 380
- Bauer, Röhmel (1995): An adaptive method for establishing a dose-response relationship. *Statistics in Medicine* 14, 1595–1607
- Proschan, Hunsberger (1995): Designed extension of studies based on conditional power. *Biometrics* 51, 1315–1324.
- Posch, Bauer (1999): Adaptive two stage designs and the conditional error function. *Biometrical Journal* 41: 689–696.
- Wassmer (1999): *Statistische Testverfahren für gruppensequentielle und adaptive Pläne in klinischen Studien*. Verlag Alexander Mönch, Köln
- Lehmacher, Wassmer (1999): *Adaptive sample size calculations in group sequential trials*. *Biometrics* 55: 1286–1290
- Brannath et al. (2002): *Recursive combination tests*. *JASA* 97 (457): 236–244
- Müller, Schäfer (2004): A general statistical principle for changing a design any time during the course of a trial. *Statistics in Medicine* 23, 2497–2508
- Bauer, König (2006): The reassessment of trial perspectives from interim data – a critical view. *Statistics in Medicine* 25, 23–36
- Vandemeulebroecke (2006): An investigation of two-stage tests. *Statistica Sinica* 16, 933–951
- Brannath, Gutjahr, Bauer (2012): Probabilistic foundation of confirmatory adaptive designs. *JASA* 107: 824-832
- Xsiao, Liu, Mehta (2019): Optimal promising zone designs. *Biometrical Journal* 61: 1175-1186

References (3)

Clinical trial examples

- Vandemeulebroecke, Bornkamp, Bretz, Pinheiro (2010): Adaptive dose-ranging studies. Chapter 11 in: Handbook of Adaptive Designs for Pharmaceutical and Clinical Development. Chapman & Hall
- Barnes et al. (2010): Integrating indacaterol dose selection in a clinical study in COPD using an adaptive seamless design. Pulmonary Pharmacology & Therapeutics 23: 165–171
- Schmoll et al. (2012): Cediranib with mFOLFOX6 versus bevacizumab with mFOLFOX6 as first-line treatment for patients with advanced colorectal cancer: A double-blind, randomized phase III study (HORIZON III). Journal of Clinical Oncology 30, 3588–3595
- Cuffe, Lawrence, Stone, Vandemeulebroecke (2014): When is a seamless study desirable? Case studies from different pharmaceutical sponsors. Pharmaceutical Statistics 13, 229–237

Software reviews

- Wassmer, Vandemeulebroecke (2006): A brief review on software developments for group sequential and adaptive designs. Biometrical Journal 48: 732–737
- Tymofyeyev (2014): A Review of Available Software and Capabilities for Adaptive Designs. Chapter in: Practical Considerations for Adaptive Trial Design and Implementation. Springer

Commercial software

- ADDPLAN: <http://www.iconplc.com/innovation/addplan/>
- EastAdapt and EastSurv: <http://www.cytel.com/software/east>

R packages

- adaptTest: <https://cran.r-project.org/web/packages/adaptTest/index.html>
- AGSDest: <https://cran.r-project.org/web/packages/AGSDest/index.html>
- asd: <https://cran.r-project.org/web/packages/asd/index.html>
- rpact: <https://www.rpact.com/>